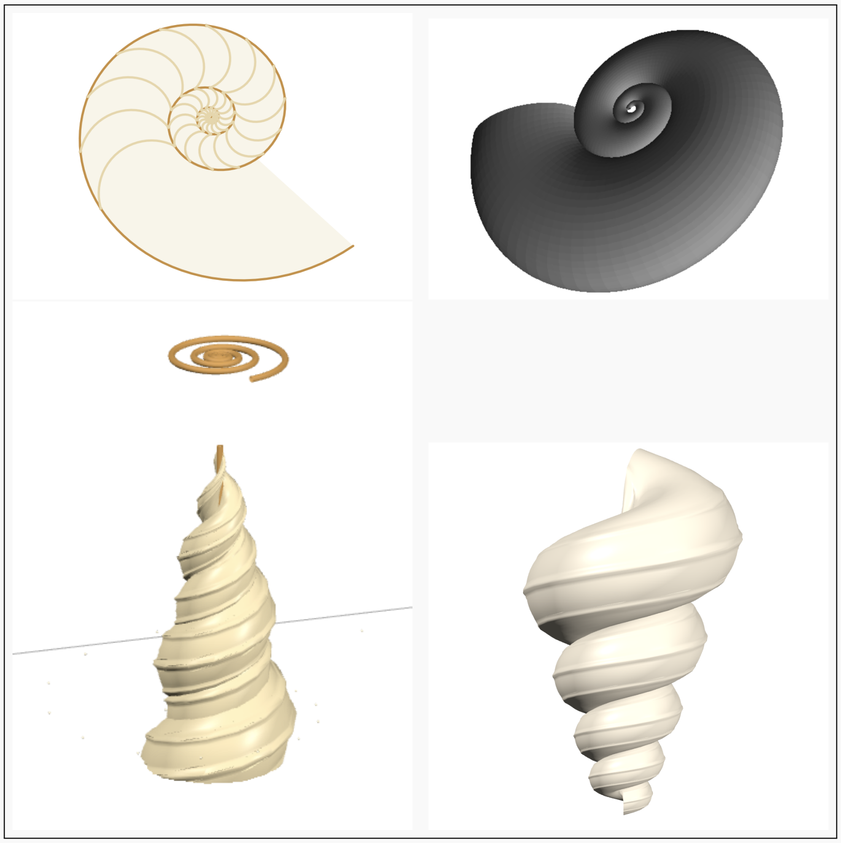
0.1 Celina 2 - Seashell Patterns

The Idea. What are the governing equations that determine the patterns and spirals of seashells, and how can we graph them?

The Story. Math is present all throughout the natural world. Shells look complicated, but many shell shapes can be built from a small number of growth rules. A shell grows by adding material at its opening. If the opening expands, rotates, and twists as it grows, the result can be a spiral, a coiled shell, or a tall 3D form. Shells follow logarithmic spirals, which keep the same shape while scaling - describing the shell's self-similar growth. We were able to use these tools to model nautilus and turritella shells in Desmos and Mathematica.

Visuals.



Try this.

- Try altering **A** and **B** to see how this affects the shape and size of the shell.
- Try adjusting parameters associated with the chamber walls to make it as realistic as possible.

Interactive Graphs.

- Nautilus Shell Desmos Graph
- Turritella Shell Desmos Graph

The Mathematics.
The basic nautilus shell spiral follows the equation

$$r(t) = Ae^{Bt} \tag{1}$$

where A sets the starting scale and B controls the growth rate (how quickly the spiral expands). In cartesian coordinates, this is

$$x(t) = r(t)cos(t) \qquad y(t) = r(t)sin(t) \tag{2}$$

The difference in growth rate between the two sides determines how tight the spiral will be. For the turritella shell, a TNB frame can be used to show the 3D path the spiral follows as it grows. An aperture circle with a changing radius that follows a helix-style path can be used to show the growth of this shell.

Comparison with the golden ratio.
While the growth model resembles the golden spiral, its growth rate is actually much smaller. For the nautilus shell model, we used $\mathbf{B} = \frac{\ln(3.2)}{2\pi} \approx \mathbf{0.185}$, whereas using the golden ratio, $\mathbf{B} = \frac{2\ln(\varphi)}{\pi} \approx 0.306$. The natural log is taken of the growth factor per every full turn. For the golden ratio, φ is the growth factor per quarter turn. For every full turn, this is $\varphi^4 = (\frac{1+\sqrt{5}}{2})^4$.

How to use this.

- These visuals show real-life representations of the Fibonacci spiral and exponents in the natural world.
- This uses Calculus III tools, such as logarithms, polar coordinates, and 3D curves.

Questions to explore / Connections.

- How else might logarithmic growth rates appear in nature, and are they more or less similar to the golden spiral?
- This project heavily relies on polar coordinates and parameterization, concepts which are further expanded on in multi-variable calculus.

Notes / Further reading.

- Interactive Webpage
- How Seashells Take Shape
- Modeling Seashells
- A Method for Quantifying, Visualising, and Analysing Gastropod Shell Form
- How Are Seashells Made