Modeling the Motion of a Curling Soccer Ball - Magnus Effect, Drag and Side-spin

Andrew Loveless, Arjav Dhorajia
Washington eXperimental Mathematics Lab
University of Washington
ardhoraj@uw.edu

***Abstract*—The main question is: How does a soccer ball curve through the air? A ball kicked towards the goal does not simply follow a parabola. It has to be modeled in 3D - pulled down by gravity, air resistance and spin can create a sideways bend through the Magnus Effect. Famous freekicks such as Beckham against Greece and Roberto Carlos' shot can be explained by a combination of variables and equations. This project investigates how calculus, differential equations and physics can be used to model and visualize the motion of the trajectory of a curling soccer ball in three dimensions.**

I. Introduction

A curling soccer free kick is one of the most striking phenomena in sports. As the ball travels through the air, several forces act on it. These forces along with projectile motion and spin, combine to produce curved trajectories. The motion of a curling ball is an excellent application of multivariable calculus, differential equations and mechanics (physics aspect). The Magnus Effect explains how rotating objects experience a force perpendicular to their direction of motion, causing the ball to bend through the air. This paper investigates the mathematical modeling of a soccer ball in a three-dimensional system of differential equations. Numerical methods and computer visualizations then simulate realistic free-kick trajectories and analyze how changing the physical parameters affects the motion.

II. Terminologies

A. Magnus Effect and Spin

The Magnus Effect is the phenomenon responsible for the curved trajectory of a spinning soccer ball. When a ball rotates through the air, the friction between the surface of the ball and the surrounding air causes the air on opposite sides of the ball to move differently. One side moves with the motion of the ball, while the other moves against the direction of motion. This creates a pressure difference, producing a sideways force known as the Magnus Force. This causes the bend/curl during flight.

Importantly, the Magnus effect does not create the spin of the ball. The spin is produced at the moment of impact between the player's boot and the ball. The angle of the kick, the position of contact relative to the center of the ball and speed determine the initial angular velocity.

III. Phenomenon Explained

The motion can be modeled as the interaction of several physical forces acting simultaneously throughout the flight of the ball. At the moment of impact, the player's boot transfers energy into both linear and rotational motion. The linear component determines the initial velocity and launch direction while the rotational component creates spin. The x- and y-axes describe horizontal movement while the z-axis represents the vertical direction (height). Once the ball leaves the foot, four forces act on it - gravity, drag, spin and the Magnus Effect. Gravity acts downward along the z-axis, drag acts opposite to the direction of motion and reduces the ball's speed. Spin is generated by the friction between the ball and the surrounding air, producing the Magnus Force, which acts perpendicular to the direction of motion, causing the ball to bend/curl.

IV. Equations and Math

A. Position, Acceleration and Velocity

Here the x-axis and y-axis represent motion in the horizontal plane while the z-axis is the vertical component.

In 3D, the ball's position is

$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle.$$

Velocity is

$$\vec{v}(t) = \vec{r}'(t) = \langle x'(t), y'(t), z'(t) \rangle.$$

Acceleration is

$$\vec{a}(t) = \vec{v}'(t) = \vec{r}''(t).$$

The speed is

$$|\vec{v}| = \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}.$$

B. Newton's Law

$\boxed{\text{motion} = \text{gravity} + \text{drag} + \text{spin.}}$

The main model is Newton's second law:

$$m\vec{a} = \vec{F}_g + \vec{F}_D + \vec{F}_M.$$

Here:

$$\vec{F}_g = \text{gravity},$$

$$\vec{F}_D = \text{drag force},$$

$$\vec{F}_M = \text{Magnus force from spin}.$$

So,

$$\vec{a} = \frac{1}{m}\left(\vec{F}_g + \vec{F}_D + \vec{F}_M\right).$$

C. Gravity

Gravity is 0 for the x and y axes components and -mg for the vertical component (z-axis)

$$\vec{F}_g = \langle 0, 0, -mg\rangle.$$

If there is no drag and no spin, then

$$\vec{a} = \langle 0, 0, -g\rangle.$$

With initial position and initial velocity the path is:

$$x(t) = x_0 + v_{x0}t,$$

$$y(t) = y_0 + v_{y0}t,$$

$$z(t) = z_0 + v_{z0}t - \frac{1}{2}gt^2.$$

D. Air Resistance

Drag acts opposite the direction of motion.

A common sports-ball model is quadratic drag:

$$\vec{F}_D = -k|\vec{v}|\vec{v}.$$

Equivalently, using physical constants:

$$\vec{F}_D = -\frac{1}{2}\rho C_D A|\vec{v}|\vec{v}.$$

Here:

ρ = air density, C_D = drag coefficient,

$A = \pi R^2$ = cross-sectional area.

Thus the drag acceleration is

$$\vec{a}_D = -\frac{k}{m}|\vec{v}|\vec{v}.$$

E. Spin and the Magnus Effect

Let

$$\vec{\omega} = \langle \omega_x, \omega_y, \omega_z\rangle$$

be the angular velocity vector.

The magnitude

$$|\vec{\omega}|$$

is the spin rate, and the direction of $\vec{\omega}$ is the spin axis.

A simplified Magnus model is

$$\vec{F}_M = c(\vec{\omega} \times \vec{v}).$$

So,

$$\vec{a}_M = \frac{c}{m}(\vec{\omega} \times \vec{v}).$$

The angular velocity vector points along the spin axis according to the right-hand rule. The cross product determines the direction of the Magnus Force. The resulting vector is always perpendicular to both the spin axis and the velocity vector. If the spin axis changes, the Magnus Force changes direction, causing the ball to bend differently during flight.

V. Initial Contact Model: Boot-Ball Interaction

The force at the moment of contact is split into:

- **Linear Impulse** - translational velocity
- **Angular Impulse** - rotational velocity (spin)

When the player's boot strikes the ball, the force vector decomposes into the two aforementioned components. The component acting primarily through the center of mass produces translational motion, and the tangential component acting off-center produces torque and angular momentum.

The impulse-momentum relation:

$$\vec{J} = \int \vec{F}\,dt = m\Delta\vec{v}$$

where:

$$\vec{J} = \text{impulse},$$

$$m = \text{mass of the ball},$$

$$\Delta\vec{v} = \text{change in linear velocity}.$$

Rotational motion is generated through torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where:

$$\vec{r} = \text{vector from the center of the ball to the contact point}.$$

The angular impulse equation becomes

$$\int \vec{\tau}\,dt = I\vec{\omega}$$

where:

$$I = \text{moment of inertia},$$

$$\vec{\omega} = \text{angular velocity vector}.$$

VI. Combined Differential Equation Model

$$\boxed{\begin{aligned} a_x &= \frac{-k|\vec{v}|v_x - c w v_y}{m}, \\ a_y &= \frac{-k|\vec{v}|v_y + c w v_x}{m}, \\ a_z &= -g - \frac{k|\vec{v}|v_z}{m}. \end{aligned}}$$

VII. Starting Constants Utilized

The following values provide realistic starting parameters for a regulation soccer ball:

$$g = 9.81 \text{ m/s}^2$$

$$m = 0.43 \text{ kg}$$

$$R = 0.11 \text{ m}$$

$$A = \pi R^2 \approx 0.038 \text{ m}^2$$

$$\rho = 1.2 \text{ kg/m}^3$$

$$C_D \approx 0.25$$

The drag constant is approximated by

$$k = \frac{1}{2}\rho C_D A$$

which gives

$$k \approx 0.0057.$$

VIII. Beckham against Greece Model

David Beckham's famous free kick against Greece in 2001 can be modeled using the system developed in this paper.

Approximate initial conditions are:

$$v_0 \approx 30 \text{ m/s}$$

$$\theta \approx 20^\circ$$

$$|\vec{\omega}| \approx 50 \text{ rad/s}$$

An off-center strike produced significant side-spin, creating a Magnus Force that bent the ball around the wall.

IX. The Knuckleball shot

So far, the paper focused on how a ball moves with spin. However, when there is hardly any spin, the ball moves differently. This motion is the knuckleball, a wobbly and unpredictable soccer shot. A ball with high spin cuts through air along a smooth curved path, with the Magnus Force acting consistently on either side of the ball. A ball with little to no spin has no such stabilizing force, making the motion wobbly. A soccer ball has seams and panels that interact differently with the airflow depending on specific orientations. An uneven pressure is created on both sides causing an imbalance that makes the ball dip and shift in random directions.

This can be modeled by adding a small random "noise" term to our original model.

$$\vec{a} = \frac{1}{m}\left(\vec{F}_g + \vec{F}_D + \vec{F}_M + \vec{F}_{\text{noise}}\right)$$

This noise term represents an unpredictable push caused by uneven airflow. It acts perpendicular to the ball's velocity, and the direction shifts throughout the flight rather than changing evenly at every instant like in the spinning motion.

X. Conclusion

This paper discussed a three-dimensional mathematical model for the flight of a soccer ball, combining gravity, quadratic drag and the Magnus Force into one system of differential equations. Based on Newton's second law, the motion was broken into two component forces at time of contact. The Beckham freekick against Greece, shows how an off-center strike generated significant side-spin, with the Magnus Force bending the ball around the wall. This paper also has an extension where different starting values and their resultant varying kicks are shown visually. The Knuckleball freekick is almost the opposite of Beckham's. It has a factor of randomness, generated by little to zero side-spin. The variations in airflow on either surfaces of the ball causes it to dip and makes the motion unpredictable. Assumptions were made to model the motion realistically. The spin rate was treated as a constant, whereas in reality it decreases with the motion. Other values were approximated in order to reflect real-life as closely as possible. Ultimately, the model shows that the complexity of a curling freekick can be explained by multivariable calculus, differential equations and physics. This model can also form the foundations of motion of other balls ranging from the tennis slice shot to the unpredictable beach volleyball.

References

[1] Mamman Haruna, Ekene, K. I., Ojo, O. O. (2025). Projectile Dynamics under Aerodynamic Drag: Application to the Flight of a Soccer Ball. Tech-Sphere Journal for Pure and Applied Sciences, 2(1). https://doi.org/10.5281/zenodo.17102567

[2] Ved Prakash. Application of Magnus Effect in Case of a Soccer Ball. Journal of Aerospace Engineering and Technology. 2016; 6(3): 1–4p

[3] The Perfect Kick — NCSA — National Center for Supercomputing Applications — Office of the Vice Chancellor of Research and Innovation — Illinois. (2025, December 10). Illinois.edu. https://www.ncsa.illinois.edu/2025/12/10/the-perfect-kick/

[4] Texier, B. D., Cohen, C., Quéré, D., Clanet, C. (2016). Physics of knuckleballs. New Journal of Physics, 18(7), 073027. https://doi.org/10.1088/1367-2630/18/7/073027

[5] Dizikes, P. (2014, June 17). Explained: How does a soccer ball swerve? MIT News — Massachusetts Institute of Technology. https://news.mit.edu/2014/explained-how-does-soccer-ball-swerve-0617

[6] [WSRP23] Parabolic perspectives: exploring the diverse trajectories of soccer balls - Online Technical Discussion Groups—Wolfram Community. (2026). Wolfram.com. https://community.wolfram.com/groups/-/m/t/2964070