Puzzle 2: The Vectors

by Jonah Ostroff

1. Compute $\langle 5, -3, 0\rangle + \langle -4, 4, 1\rangle$.
2. Compute $2\langle -6, 5, 9\rangle - 3\langle -4, 3, 6\rangle$.
3. Compute $(\mathbf{i} \cdot \mathbf{j})\mathbf{k} + (\mathbf{i} \cdot \mathbf{i})\mathbf{i}$.
4. If $\mathbf{a} = \langle 4, 2, -3\rangle$, $\mathbf{b} = \langle 2, 2, 4\rangle$, and $\mathbf{c} = \langle 6, 1, 3\rangle$, compute $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$.
5. If $\mathbf{a} = \langle -1, -1, -1\rangle$ and $\mathbf{b} = \langle 2, 1, 0\rangle$, compute $\text{proj}_{\mathbf{a}}(\mathbf{b})$.
6. If $\mathbf{a} = 7\mathbf{i}$ and $\mathbf{b} = \mathbf{i} - 8\mathbf{j}$, compute $\text{proj}_{\mathbf{a}}(\mathbf{b})$.
7. If $\mathbf{a} = \langle 4, -8, 4\rangle$ and $\mathbf{b} = \langle 3, -3, 3\rangle$, compute $\mathbf{b} - \text{proj}_{\mathbf{a}}(\mathbf{b})$.
8. Find a unit vector perpendicular to both $\langle 0, 6, 3\rangle$ and $\langle 0, 2, 3\rangle$.
9. Compute $(7\mathbf{j} + 4\mathbf{k}) \times (5\mathbf{j} + 3\mathbf{k})$.
10. Compute $\big((\mathbf{i} \times \mathbf{j}) \times \mathbf{j}\big) \times \mathbf{k}$.
11. Give a normal vector to the plane $x + y + z = 0$.
12. Give a normal vector to the plane $y = 8$.
13. Find a normal vector to the plane containing the points $(0, 0, 0)$, $(1, 7, -1)$, and $(4, 2, -4)$.
14. Find a normal vector to the plane containing the points $(0, 4, 6)$, $(1, 3, 2)$, and $(5, -1, 2)$.
15. Find a direction vector of the intersection of the planes $5x + 8z = 2$ and $2x + 3z = 0$.
16. Find a direction vector of the intersection of the planes $3y - 4z = 5$ and $-2y + 3z = 6$.
17. If $\mathbf{r}(t) = \langle \arctan(t), \cos(t), e^t - t\rangle$, find $\mathbf{r}'(0)$.
18. If $\mathbf{r}(t) = \langle 4\sqrt{t}, t^2 - 8t, \cos(t - 4)\rangle$, find $\mathbf{r}'(4)$.
19. If $\mathbf{r}'(t) = \langle 1, t, \frac{1}{t-1}\rangle$ and $\mathbf{r}(4) = \langle 3, 6, \ln(3) + 1\rangle$, find $\mathbf{r}(2)$.
20. If $\mathbf{r}'(t) = \langle \cos(t), 2t, te^t\rangle$ and $\mathbf{r}(\pi) = \langle 0, \pi^2, (\pi - 1)e^\pi + 1\rangle$, find $\mathbf{r}(0)$.
21. If $\mathbf{r}(t) = \langle 0, \cos(\pi + t), \sin(\pi + t)\rangle$, find $\mathbf{N}(0)$.
22. If $\mathbf{r}(t) = \langle 0, \cos(\pi + t), \sin(\pi + t)\rangle$, find $\mathbf{B}(0)$.

The second thing you should know about the Secret Multivariable Puzzle Challenge is that the prize is a multivariable function. If x other people finish the challenge and y of them complete it before you, you earn a bonus of

$$\frac{(x + 50)\,(y + 40)}{(4x + 100)\,(y + 5)}\%$$

to your overall score. If you complete the puzzle with a team, you'll each get the average of the number of points you would have earned separately. This boost is added after the curve is set for the class as a whole, so you won't be penalized for not participating.

If you can't see what those problems have to do with the answer, this grid might help.

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	21.	22.
i	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
j	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
k	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○