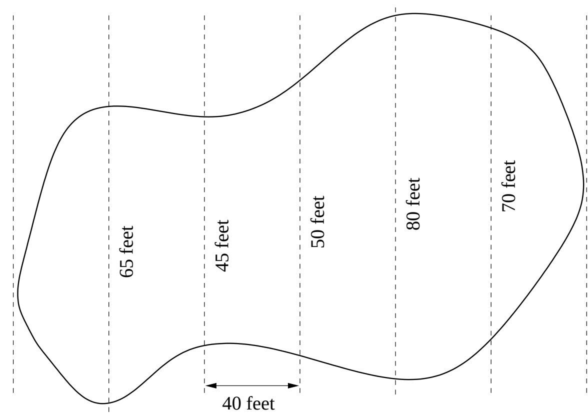
In this work sheet we'll study the problem of finding the area of a region bounded by curves. We'll first estimate an area given numerical information. The we'll use calculus to find the area of a more complicated region.

The Lake

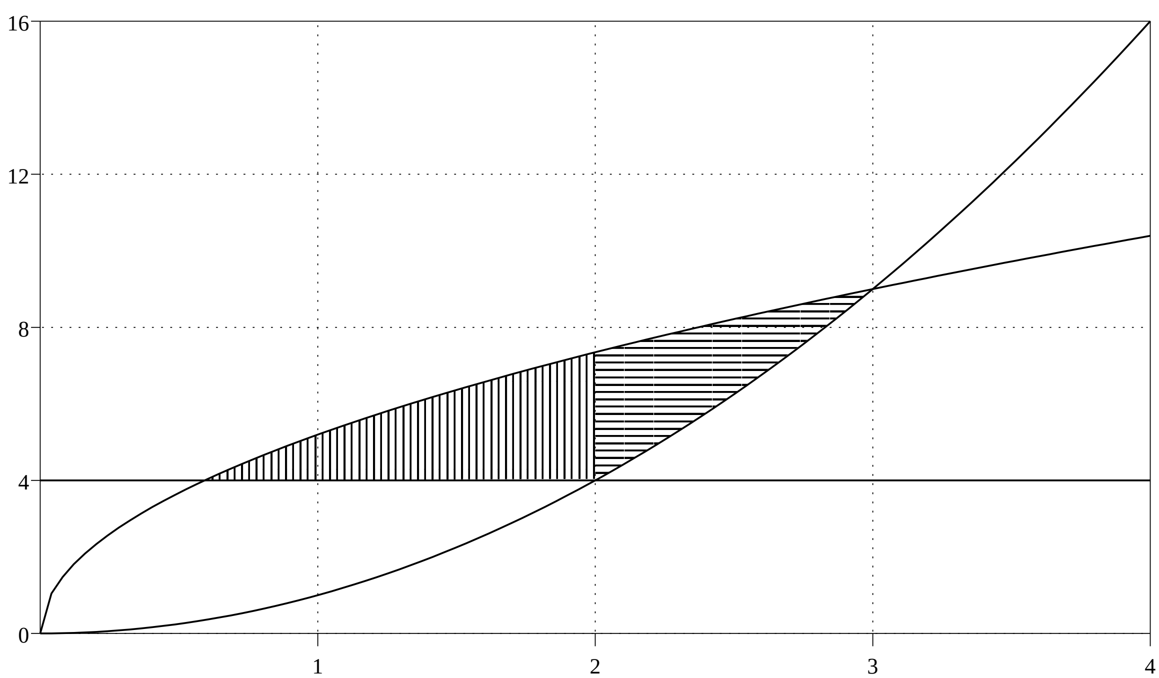
1 The widths, in feet, of a small lake were measured at 40 foot intervals. Estimate the area of the lake.



I can think of three possible methods:
Left Endpoints: $(0 \times 40) + (65 \times 40) + (45 \times 40) + (50 \times 40) + (80 \times 40) + (70 \times 40) = 12400\ ft^2$
Right Endpoints: $(65 \times 40) + (45 \times 40) + (50 \times 40) + (80 \times 40) + (70 \times 40) + (0 \times 40) = 12400\ ft^2$
Midpoints: $(65 \times 80) + (50 \times 80) + (70 \times 80) = 14800\ ft^2$

Area Bounded by Three Curves

2 On the grid below sketch the graphs of $y = 4$, $y = x^2$ and $y = \sqrt{27x}$. (The last one is just a piece of a sideways parabola).



3 Shade the "triangular" region bounded by the graphs of the three functions that lies above the horizontal line.

4 Compute the x-coordinate of the left endpoint of the region.

$4 = \sqrt{27x}$ *gives* $x = \frac{16}{27}$

5 Compute the x-coordinate of the right endpoint of the region.

$x^2 = \sqrt{27x}$ *gives* $x = 3$

6 Note that the top of the region consists of a single curve, but the bottom of the region consists of two differentcurves. Find the x-coordinate where these two curves meet.

$4 = x^2$ *gives* $x = 2$, *since we know* x *is positive*

7 Sketch in a vertical line at the x-coordinate you found in the last problem. This divides the region into two smaller sub-regions.

8 Compute the area of the left sub-region.

$$\int_{\frac{16}{27}}^{2} \sqrt{27x} - 4\, dx = 4\sqrt{6} - \frac{584}{81} \approx 2.588$$

9 Compute the area of the right sub-region. Add the two areas together to get the total area.

$$\int_{2}^{3} \sqrt{27x} - x^2\, dx = \frac{35}{3} - 4\sqrt{6} \approx 1.8687$$

$$\left(4\sqrt{6} - \frac{584}{81}\right) + \left(\frac{35}{3} - 4\sqrt{6}\right) = \frac{361}{81} \approx 4.4568$$

10 Recompute the area using the following trick. Solve for x as a function of y in the two non-constant functions. Find the area by integrating with respect to y. Is this easier?

$$x = \sqrt{y} \qquad x = \frac{y^2}{27}$$

$$\int_{4}^{9} \sqrt{y} - \frac{y^2}{27}\, dy = \frac{361}{81}$$

This seems a lot easier to me!