The following problems should help you review for the final exam. Don't hesitate to ask for hints if you get stuck.

Arclength and Approximation

1.   Write an integral that computes the arclength of the curve $y = e^{x/2}$ between $x = 0$ and $x = 2$. Use Simpson's Rule with $n = 4$ subintervals to estimate the value of the integral.

Center of Mass

2.   Find the center of mass of a plate with constant density that occupies the region $-\frac{1}{4}\pi \le x \le 0, \quad 0 \le y \le \sec^2 x.$
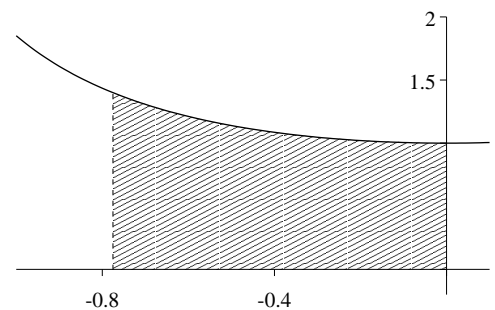
Net and Total Distance

3.   You throw a ball straight up into the air with velocity 40 ft/sec and catch it (at the same height) when it comes back down. What is the total distance traveled by the ball?

Differential Equations

4.   Let $f(t)$ be a continuous function and let a be a constant. Show that $y = e^{-at} \int_0^t e^{as} f(s)\, ds$ satisfies the differential equation $\dfrac{dy}{dt} + ay = f(t)$.

5.   An electric circuit with resistance 10 ohms and inductance 2 henrys is powered by a 12-volt battery. The current I (in amperes) at time t (in seconds) in such a circuit satisfies the differential equation

$$2\frac{dI}{dt} + 10I = 12.$$

Suppose that $I = 0$ when the circuit is activated at time $t = 0$.
(a) Find the current I at all times $t > 0$.
(b) Find the limiting value of I as $t \to \infty$.
(c) After what time is the current within 0.1 ampere of its limiting value?