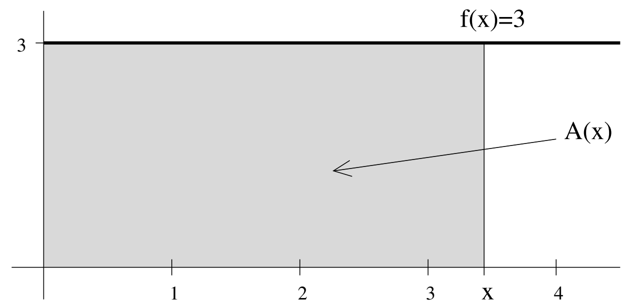
Name ______________________ Quiz Section ______

Introduction

Many interesting and useful functions can be defined as the area under some other function. There is a very nice relationship between the original function and the area function. We will explore that relationship in this worksheet.

Area Functions

1a Define $A(x)$ to be the **area** bounded by the x-axis and the function $f(x) = 3$ between the y-axis and the vertical line at x. (See the diagram.)



$A(1) =$ __________ $A(2) =$ __________

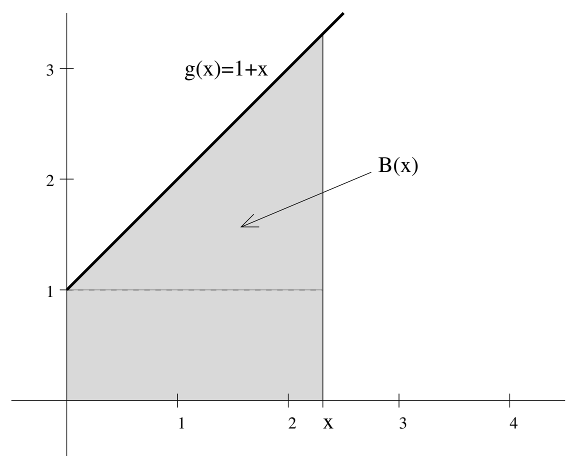
$A(3) =$ __________ $A(4) =$ __________

and, in general,

$A(x) =$ ____________________ (a formula)

Shade the region whose area is $A(3) - A(1)$.

1b Define $B(x)$ to be the **area** bounded by the x-axis and the function $g(x) = 1 + x$ between the y-axis and the vertical line at x. (See the diagram.)



$B(1) =$ __________ $B(2) =$ __________

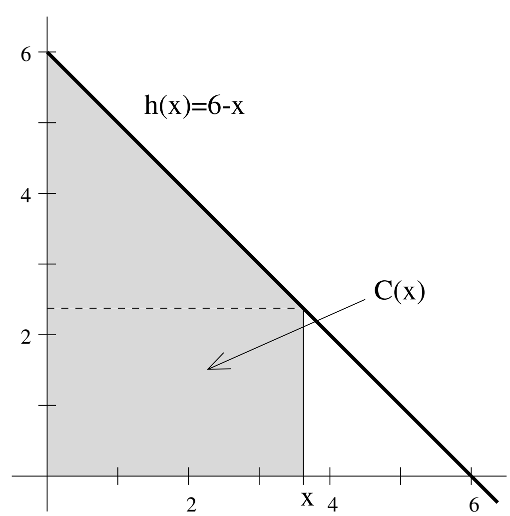
$B(3) =$ __________ $B(4) =$ __________

and, in general,

$B(x) =$ ____________________ (a formula)
(Hint: think triangle + rectangle)

Shade the region whose area is $B(3) - B(1)$.

1c Define $C(x)$ to be the **area** bounded by the x-axis and the function $h(x) = 6 - x$ between the y-axis and the vertical line at x. (See the diagram.)



$C(1) =$ __________ $C(2) =$ __________

$C(3) =$ __________ $C(4) =$ __________

and, in general,

$C(x) =$ ____________________ (a formula)

Shade the region whose area is $C(3) - C(1)$.

For each of the above, the **area** increases as x increases. So $A(x)$, $B(x)$ and $C(x)$ are increasing functions even though $f(x)$ is constant, $g(x)$ is increasing and $h(x)$ is decreasing. (There is a difficulty with $C(x)$ when x gets larger than 6. We'll deal with that later.)

1d Now calculate the derivatives of the area functions from problems 1, 2 and 3 above:

$A'(x) =$ ________________ $B'(x) =$ ________________ $C'(x) =$ ________________

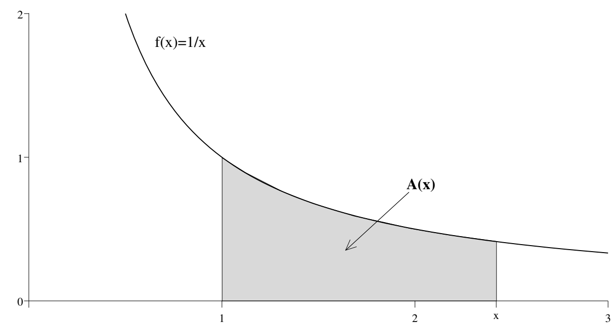
How is $A'(x)$ related to $f(x)$ in problem 1?

How is $B'(x)$ related to $g(x)$ in problem 2?

How is $C'(x)$ related to $h(x)$ in problem 3?

The Natural Logarithm

2a Define $A(x)$ to be the **area** bounded by the x-axis and the function $f(x) = 1/x$ between the line $x = 1$ and the vertical line at x. (See the diagram.)



Based on your work in problem 1,

$A'(x) =$ ____________

Compute $A(1) =$ ____________

Compute $A(x) =$ ____________

2b So the area under $f(x) = 1/x$ between $x = 1$ and $x = 2$ is equal to $\ln(2)$. Outline this area on the graph. We'll use estimates of this area to compute approximations of $\ln(2)$.

2c Slice the area up into 4 pieces by drawing 3 evenly spaced vertical lines from the x-axis up to the curve.

2d Using the left side of each slice as the height, sketch in 4 rectangles on your graph. What are the x-coordinates of the sides of the rectangles? Plug these x-coordinates into $f(x) = 1/x$ to compute the heights of the rectangles. Find the areas of the 4 rectangles and add them up. This is your first approximation of the area under the curve, and $\ln(2)$. Is it an over-estimate or an under-estimate?

2e Using the right side of each slice as the height, sketch in 4 rectangles on your graph. Find the area of these rectangles and add them up. This is your second approximation of the area under the curve, and $\ln(2)$. Is it an over-estimate or an under-estimate?

2f Take the average of your two estimates to get a new estimate of $\ln(2)$. How does it compare with the value given by your calculator?

2g Use the midpoint of each slice to determine the height and sketch in the resulting 4 rectangles. Use them to approximate $\ln(2)$. Can you tell if you are getting an over-estimate or and under-estimate? Which of your four estimates gives you the closest answer to the value given by your calculator?