Math 125 **Final Exam** **Winter 2018**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and stow away all cell phones, watches, pagers, music players, and other similar devices.

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.

- You can use only a Texas Instruments TI-30X IIS calculator.

- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.

- You may use directly the integral formulas # 1-18 in the table from section 7.5 of your textbook (posted on the departmental math 125 website), without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.

- Place $\boxed{\text{a box around your answer}}$ to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\dfrac{\pi}{3}$ or $5\sqrt{3}$).

- If you need more room, use the backs of the pages and indicate that you have done so.

- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (10 points) Evaluate the following integrals. Show your work, simplify, and box your answers.

(a) $\int \sqrt{x}\,\ln(x)\,dx$

(b) $\int \dfrac{1}{x^2\sqrt{4+x^2}}\,dx$

2. (10 points) Evaluate the following integrals. Show your work, simplify, and box your answers.

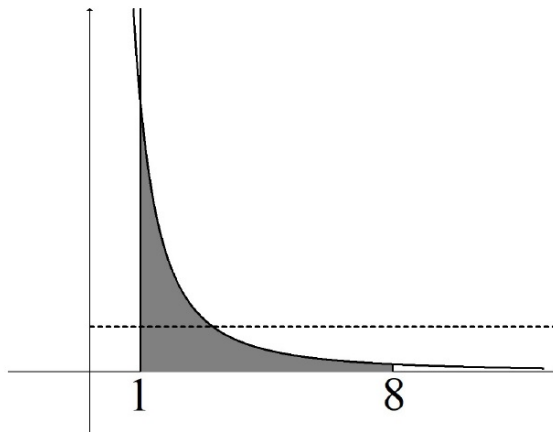(a) $\displaystyle\int \frac{x^2-x+2}{x^3+2x}\,dx$

(b) $\displaystyle\int_1^3 \left(\frac{x+1}{x}\right)^2 - |x-2|\,dx$

3. (10 points) For each of the two **improper** integrals below, if the integral converges, evaluate it and if it diverges, explain why. Show all limit computations.

(a) $\displaystyle\int_0^3 \frac{x}{3-x}\,dx$

(b) $\displaystyle\int_0^\infty xe^{-3x}\,dx$

4. (10 points) Consider the region R bounded by $y = 0$, $x = 1$, $x = 8$, and $y = \dfrac{16}{x^2}$.

   (a) (4 points) Find the area of R.

   (b) (6 points) Find the value of b such that the ***horizontal*** line $y = b$ (the dotted line in the figure above) bisects R into two regions of equal area. You may give your answer either in exact form, or in decimal form accurate to at least two digits after the decimal point.

5. (10 points) The acceleration of a particle that moves on a straight line during $0 \le t \le 3$ is given by the graph below.

   Find the **velocity** and the **position** of the particle at time $t = 3$, if the particle started from rest at position $x = 1$ at time $t = 0$.
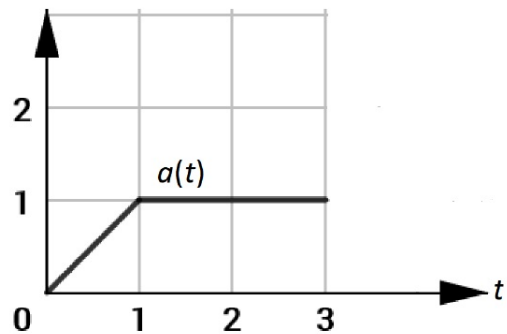
6. (10 total points) Parts (a) and (b) of this question are unrelated.

    (a) (4 points) Set up (but **DO NOT EVALUATE**) a definite integral equal to the arc length of the curve:
$$y = e^{-x^2}, \qquad 0 \le x \le 3.$$

    (b) (6 points) A CAT scan shows cross-sections (slices) of a patient's liver, spaced 1.5 cm apart. The patient's liver is 15 cm long and the cross-sectional areas, in square centimeters, are:

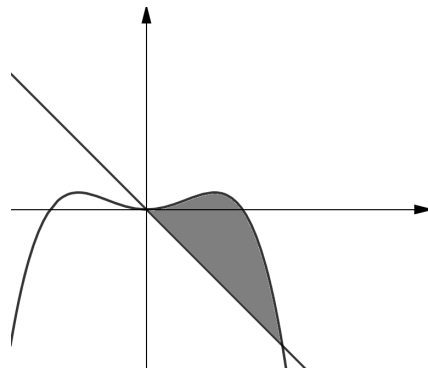$$0, 19, 57, 79, 95, 106, 116, 127, 62, 38, \text{ and } 0.$$

    Estimate the volume of the liver using the Midpoint Rule with $n = 5$ subintervals.

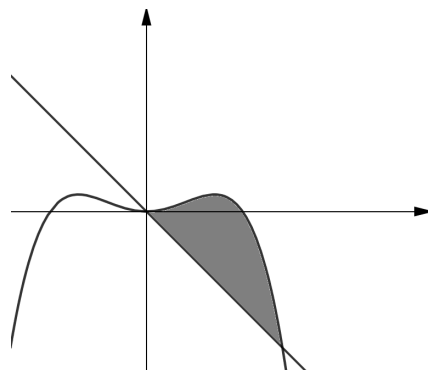7. (10 total points) Let R be the region bounded by the curves

$$y = x^2 - \frac{1}{2}x^4 \qquad \text{and} \qquad y = -2x.$$

(Note that these curves intersect at $x = 0$ and $x = 2$.)

(a) (6 points) Find the volume of the solid of revolution obtained by rotating R about the y-axis.

(b) (4 points) Set up (but **DO NOT EVALUATE** or simplify) a definite integral equal to the volume obtained by rotating the same region R about **the <u>horizontal</u> line y = 2.**

8. (10 points)   A water tank is 6 feet long, with two rectangular sides and two trapezoidal ends. Its trapezoidal ends have a height of 3 feet, and bases of 2 feet and 6 feet, as in the picture.

   The tank is filled with water to a level of 1 foot from the bottom. Water weighs 62.5 lb/ft^3.

   SET UP (but **DO NOT EVALUATE** or simplify) a definite integral equal to the work (in ft-lb) required to pump all the water to the top of the tank.
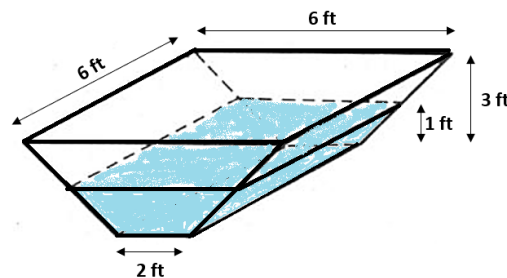
9. (10 points) Find the solution to the differential equation

$$y' = \frac{xy^3}{\sqrt{1+x^2}}$$

that satisfies the initial condition

$$y(0) = -1.$$

Give your solution in explicit form, $y = f(x)$.

10. (10 total points) A 50-gallon tank initially contains 20 gallons of water in which 10 pounds of salt are dissolved. Pure water enters the tank at a rate of 4 gal/min. Simultaneously, a drain is open at the bottom of the tank, allowing the salt-water solution to leave the tank at a rate of 2 gal/min. The solution is kept thoroughly mixed.

   (a) (2 points) Find a formula for the volume $V(t)$ (in gallons) of the salt-water solution in the tank at time t (in minutes).

   (b) (3 points) Write a differential equation for the amount $y(t)$ (in lbs) of salt in the tank at time t.

   (c) (3 points) Solve this differential equation and use the initial amount of salt in the tank to find a formula for $y(t)$.

   (d) (2 points) What is the amount of salt in the tank at the moment that the tank becomes full?