Math 125 **Final Examination** **Winter 2015**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- **No calculators** of any kind are allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (10 total points) Evaluate the following indefinite integrals.

   (a) (5 points) $\int \left(1+\tan^2\theta\right)^2 \, d\theta$

   (b) (5 points) $\int \frac{x}{\left(x^2+4x+13\right)^{3/2}} \, dx$

2. (10 total points) Evaluate the following definite integrals.

   (a) (5 points) $\int_3^4 x\sqrt{4-x}\,dx$

   (b) (5 points) $\int_0^2 \frac{4-2x}{x^2+4x+3}\,dx$

3. (10 total points) Consider the improper integral

$$\int_0^\infty x e^{px}\, dx,$$

where p is a constant.

(a) (4 points) Find the values of p for which the improper integral converges.

(b) (6 points) Evaluate the integral for those values of p.

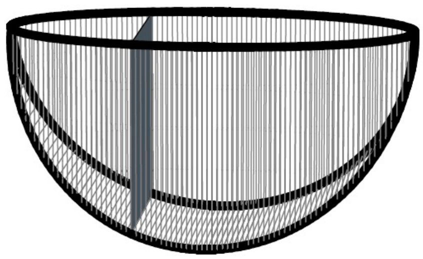
4. (10 points) A region is bounded on the top by the curve $y = \sqrt{6 + \cos(x)}$ and on the bottom by the x-axis. On the left it is bounded by a vertical barrier which at a given instant is at $x = -\pi$ and is moving to the left at α units/sec. On the right it is bounded by a vertical barrier which at that instant is at $x = \pi$ and is moving to the right at β units/sec. Find the instantaneous rate (in units^2/sec) at which area is being uncovered at that instant. Your answer should involve α and β.

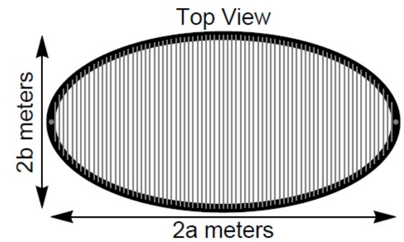
5. (10 points) A particle is moving along the x-axis. For time $t \geq 0$ seconds, its acceleration is given by

$$a(t) = 2t - 3 \quad \mathrm{m/sec}^2,$$

and its initial velocity is $v(0) = -4$ m/sec. What is the *total distance* that the particle has traveled from time $t = 0$ to the time two seconds after the positive time when the particle reverses direction?

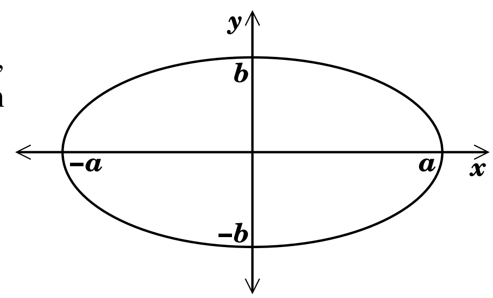
6. (10 total points) The goal of this problem is to compute the volume of the container pictured below on the left. The top rim of the container is an ellipse with the dimensions indicated in the picture on the right. Cross-sections of the container perpendicular to the long axis of the elliptical rim are squares (a representative cross-section is shown).





Recall that the equation of an ellipse centered at the origin, symmetric with respect to both the x- and y-axes, and with x-intercepts $x = \pm a$ and y-intercepts $y = \pm b$, is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$



(a) (5 points) Express the volume of the container as a definite integral.

(b) (5 points) Evaluate the definite integral to compute the volume of the container.
Your answer should involve a and b.

7. (10 points) Let $f(x)$ be a continuous function defined on the closed interval $[-2,7]$. Use the Trapezoid rule with $n=3$ to find an approximate expression for the integral

$$\int_{-2}^{7} \frac{1}{1+(f(x))^2}\, dx.$$

Your answer will involve some specific values of the function $f(x)$. If, for example, you need the value of $f(x)$ at $x=4$, write "$f(4)$" in your answer.

8. (10 points) Find the x-coordinate $\bar{x}$ of the center of mass of the uniform flat plate of density $\rho = 1$ bounded by the curve $y = \dfrac{\ln(x)}{x^2}$ and the x-axis, and between $x = 1$ and $x = e$.

9. (10 points) Find the solution of the initial value problem

$$\frac{dy}{dt} = (y + 2\sqrt{y})\,\cos^2(t), \quad y(0) = 1.$$

Give your answer in the form $y = f(t)$.

10. (10 total points) At time $t = 0$, a 20-liter container filled with water has 400 gm of salt dissolved in it. Salt is being poured in at the top at the rate of 4 gm/min. In addition, water having a salt concentration of 3 gm/liter is entering the container at 2 liters/min, and the mixed water/salt solution leaves the container at the same rate of 2 liters/min. Assume that the salt is immediately well mixed in the water. Let $y = y(t)$ denote the amount of salt (in grams) in the container at time t (in minutes).

    (a) (3 points) Set up a differential equation for $y(t)$.

    (b) (5 points) Solve the differential equation in part (a), using the initial condition. Please show all your steps carefully. You should end up with a formula for $y(t)$.

    (c) (2 points) Find $\lim_{t\to\infty} y(t)$.