Your Name

Student ID Number

Profesor's Name

Quiz Section

- Turn off and stow away all cell phones, watches, pagers, music players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- In order to receive credit, you must **show your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas in the table below. You must **show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$	**2.** $\int \frac{1}{x}\,dx = \ln\lvert x\rvert$
3. $\int e^x\,dx = e^x$	**4.** $\int b^x\,dx = \frac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$	**6.** $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$	**8.** $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$	**10.** $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert$	**12.** $\int \csc x\,dx = \ln\lvert \csc x - \cot x\rvert$
13. $\int \tan x\,dx = \ln\lvert \sec x\rvert$	**14.** $\int \cot x\,dx = \ln\lvert \sin x\rvert$
17. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	**18.** $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

- Place $\boxed{\text{a box around your answer}}$ to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- All pages are double-sided except for this cover page and the last page. You may use the blank pages for extra room but, if you want us to grade these spare pages, clearly **indicate in the problem area** that your work is on the back of the cover page or on the blank page(s) at the end of the exam.
- This exam has 10 problems on 10 pages. When the exam starts, check that your exam is complete. Good luck!

1. (10 points) Evaluate the following integrals. Show your work. Simplify and box your answers.

   (a) $\int \cos^3(t) + \tan^2(t)\sec^4(t)\,dt$

   (b) $\int_e^{e^2} \frac{1}{x(\ln(x))^2}\,dx$

2. (10 points) For each of the following integrals, choose the method that will work best on that integral. No need to justify or to compute anything. Just shade one (and only one) square next to your answer.

(a) $\int \cos^4(x)dx$

- ☐ substitution with $u = \cos^2(x)$
- ☐ replace $\cos^2(x)$ with $1 - \sin^2(x)$, then substitution with $u = \sin(x)$
- ☐ apply the half-angle formula twice
- ☐ substitution with $u = \cos(x)$

(b) $\int \frac{2x-6}{x^2-6x+12}dx$

- ☐ substitution with $u = x^2 - 6x + 12$
- ☐ factor the denominator and use partial fractions
- ☐ integration by parts
- ☐ inverse trig substitution with $x - 3 = \sec(\theta)$

(c) $\int \frac{2}{(x^2-1)^{1/2}}dx$

- ☐ substitution with $u = x^2 - 1$
- ☐ inverse trig substitution with $x = \sin(\theta)$
- ☐ factor the denominator and use partial fractions
- ☐ inverse trig substitution with $x = \sec(\theta)$

(d) $\int e^{\sqrt{x}}dx$

- ☐ substitution with $u = \sqrt{x}$, then integration by parts
- ☐ integration by parts with $u = e^{\sqrt{x}}$ and $dv = dx$
- ☐ substitution with $u = x$
- ☐ integration by parts with $u = 1$ and $dv = e^{\sqrt{x}}$

(e) $\int \frac{2x}{x^2-9x+2}dx$

- ☐ substitution with $u = x^2 - 9x + 2$
- ☐ integration by parts
- ☐ factor the denominator and use partial fractions
- ☐ substitution with $u = x$

3. (a) (6 points) Evaluate $\int x \arctan(x) dx$.

(b) (4 points) Is the improper integral $\int_0^\infty x \arctan(x) dx$ convergent or divergent? If it is divergent, justify by showing all limit calculations. If it is convergent, find the value to which it converges.

4. (10 points) You have the following data about the values of a function $y = f(x)$ and its derivative $\frac{dy}{dx} = f'(x)$ at certain values of x in the interval [0, 2]. Use this table to answer the questions below. Show your setup and your work.

x	**0**	**0.25**	**0.5**	**0.75**	**1**	**1.25**	**1.5**	**1.75**	**2**
$f(x)$	1	0.9	0.8	0.5	0	-0.4	-0.7	-0.3	1
$f'(x)$	**0**	**-0.3**	**-1**	**-1.7**	**-2**	**-1.5**	**0**	**3**	**8**

(a) Estimate the average value of f over the given interval $[0,2]$ using **Simpson's Rule** with $n = 8$ subintervals. Round your answer to the nearest 3 digits.

(b) Set up an integral equal to the arc length of $y = f(x)$ over the same interval $[0,2]$, then estimate that integral using the **Midpoint Rule** with $n = 4$ subintervals. Simplify your answer but leave it in exact form.

5. You have the following data about the values of a function $y = f(x)$ and its derivative $f'(x)$ at certain values of x in the interval [0, 2]. Use this table to answer the questions below.

x	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
$f(x)$	1	0.9	0.8	0.5	0	-0.4	-0.7	-0.3	1
$f'(x)$	**0**	**-0.3**	**-1**	**-1.7**	**-2**	**-1.5**	**0**	**3**	**8**

(a) (3 points) Evaluate $\int_0^{0.75} f'(x)dx$. Do not approximate, find the exact value of this integral.

(b) (4 points) Define $F(x) = \int_0^{f(x)} e^{t^2} dt$. Evaluate $F'(0.75)$. Do not approximate, find the exact value of this derivative.

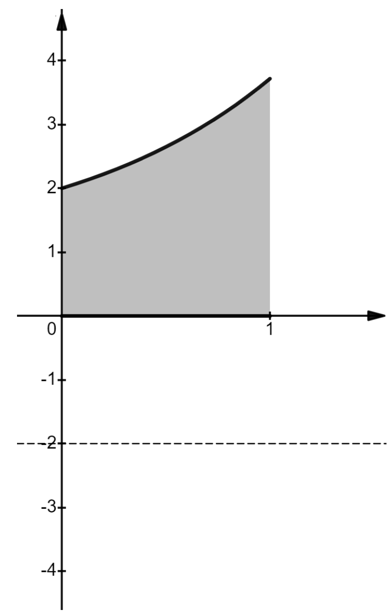
(c) (3 points) Compute $\int_0^1 xf''(x)dx$. Do not approximate, calculate the exact value of this integral.

6. (10 points) A leaky bucket was raised 7 meters. Water was leaking at a constant rate and the bucket was being lifted at a constant speed. At the start, the total mass of the bucket and water was 20 kg. At the top, the mass of the bucket and water was 6 kg. Find the total amount of work that was done.

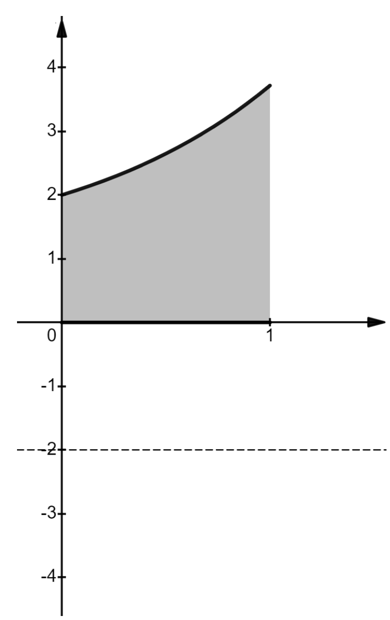
   The mass of the rope was negligibly small, so ignore the rope. Use $g = 9.8$ m/s^2.

7. (10 points) A solid of revolution is obtained by rotating the region shown below about the horizontal line $y = -2$. The region is above the x-axis, and below the graph of $y = 1 + e^x$, between $x = 0$ and $x = 1$.

   (a) Use the method of **disks/washers** to set up an integral expression equal to the volume of this solid. Do not compute or simplify the integral(s).



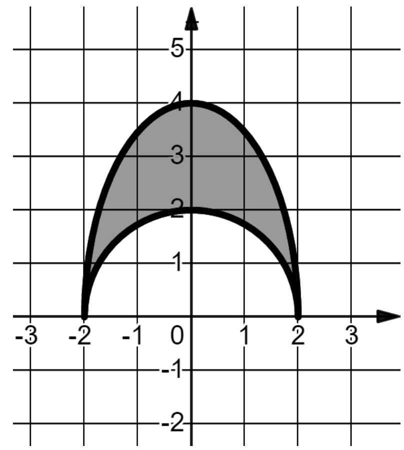
   (b) Use the method of **cylindrical shells** to set up an integral expression for the volume of the same solid. Do not compute or simplify the integral(s).



8. (10 points) Find the coordinates $(\bar{x}, \bar{y})$ of the center of mass of a flat plate occupying the shown shaded area in the xy-plane. The region is described by:

$$\sqrt{4-x^2} \leq y \leq 2\sqrt{4-x^2}$$

You must justify your answers with computations or an explanation.



9. (10 points) Solve the initial value problem. Give your answer in explicit form, $y = f(x)$.

$$\frac{dy}{dx} = \frac{1}{(\tan^2(y)+1)\left(\frac{1}{x}+1\right)} \qquad y(0) = \pi/4$$

10. A population of mountain goats in a certain region satisfies the differential equation:

$$\frac{dP}{dt} = 3P - \frac{P^2}{1000}$$

where $P = P(t)$ is the number of goats in the region after t years.

(a) (2 points) Find the carrying capacity of the region.

(b) (8 points) Solve the differential equation and determine the number of goats in the region after 2 years if the initial population was 1000 goats. Round your answer to the nearest whole number of goats.