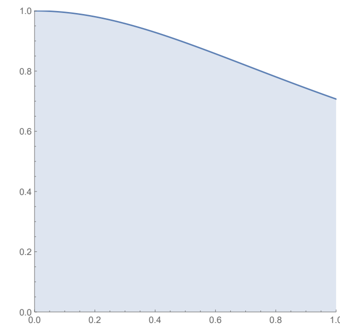
Please READ carefully these EXAM RULES:

- This exam is open textbook and open notes. You may only use class notes and your textbook during the exam.
- You may use a scientific calculator during the exam, but NOT a graphing or integrating one.
- If you have questions during the exam, you may ask your TA via Zoom.
- You may not use notes or resources from other classes.
- You may **not** get help from anyone during the exam. You may not discuss the exam with anyone until tomorrow.
- You may **not** use search engines, tutors, or any kind of apps or websites that can do Calculus during the exam.
- You may use directly any of the integrals in the table of formulas at https://sites.math.washington.edu/ m125/. Any other integrals need to be solved from these, via the methods studied in this class, showing all the steps.
- Unless otherwise instructed, make sure to **show all your work** and fully justify your answers, using the methods of this class. If the work shown is incomplete, incorrect, or unreadable, you may receive little credit, even if the answer happens to be correct.
- Simplify your answers as much as possible but leave them in exact form (e.g. $\pi\sqrt{2}+\frac{1}{2}$), unless otherwise instructed. Place a box around $\boxed{\text{your final answer}}$ to each question.
- Do not write or print two-sided. The backside of the paper will show through the page and make your solution hard to read. If we cannot read your solution, we cannot give you credit for it.
- Read each question carefully, before and after answering it. Determine how the given question relates to what you learned. Work at a steady pace, show your steps, and do your best.
- The exam is 10 pages long, and each page is worth 10 points. You have 170 minutes (2 hrs 50 min).
  Do not upload this page.

Good luck!

1. (10 points) Find the center of mass $(\bar{x}, \bar{y})$ of a metal plate bounded by the graph of the function $f(x) = \dfrac{1}{\sqrt{1+x^2}}$ on the top, the x-axis on the bottom, the y-axis on the left and the line $x = 1$ on the right.

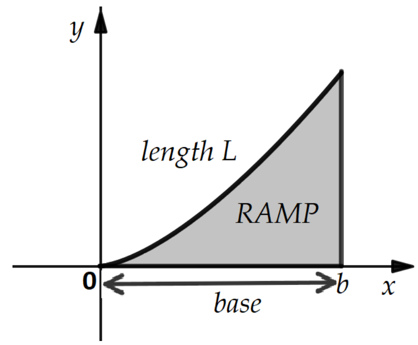
   Show all work and box your answer.



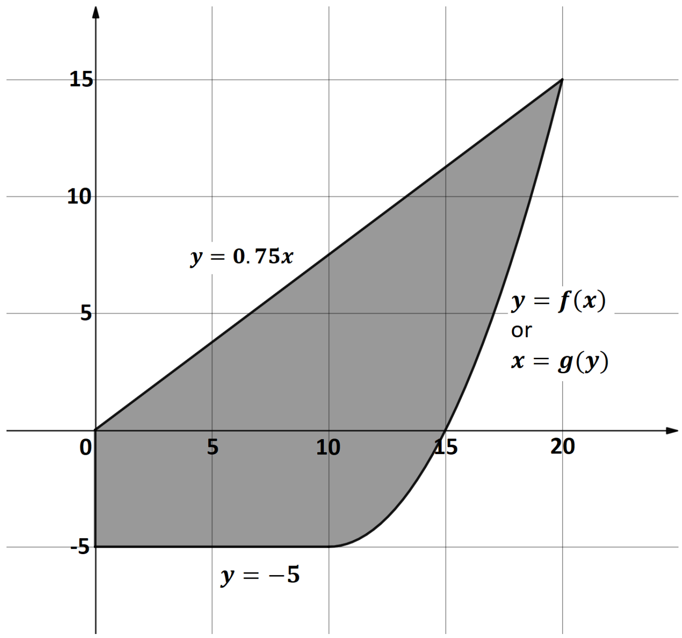
2. (10 points) While staying home in quarantine, you decide to build yourself a skateboard ramp.

   The top of the ramp is in the shape of the function $y = (4/3)x^{3/2}$, where x is the horizontal distance along the base of the ramp, in meters.

   If you want the top of the ramp to have an arclength $L = 2$ meters, how long will the base b have to be?



3. (10 points) The shaded region shown is bounded on its top by the line $y = 0.75x$, and on its bottom by $y = -5$ for $0 \leq x \leq 10$ and by some non-linear function $y = f(x)$ (with inverse $x = g(y)$) for $10 \leq x \leq 20$.



(a) Set up an integral expression in x equal to the area of this region.
Your answer will include $f(x)$. Do not simplify or compute, and no need to justify.

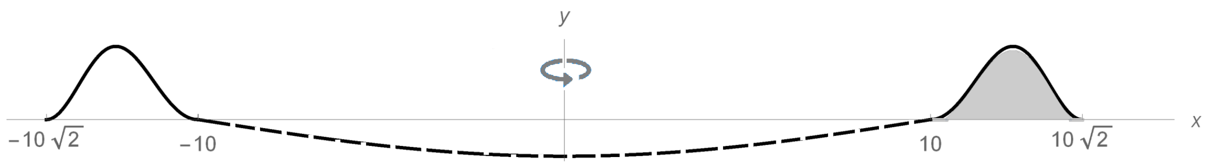
(b) Set up an integral expression in y equal to the area of this region.
Your answer will include $g(y)$. Do not simplify or compute, and no need to justify.

4. (10 points) Archeologists found an ancient circular mound created by soil dug up from the center of the structure. The mound is a solid of revolution obtained by rotating the shaded area shown below around the line at the center. Measurements show that the top of the shaded section of the mound is approximately given by the function:

$$f(x) = 2\sin^2\left(\frac{\pi x^2}{100}\right), \ \ 10 \leq x \leq 10\sqrt{2}$$

where all dimensions are in meters, and x is the distance from the center.

Compute the volume of the mound. Show all work and leave your answer in exact simplified form.



5. (10 points) A leaky bucket weighs 2 kg and initially contains 20 kg of water.

   It is being pulled up on a rope at the speed of 2 m/s while losing 1 kg of water per second. Ignore the weight of the rope and assume that the gravitational acceleration is 9.8 m/s^2.

   The work done to lift the bucket to the height h above the initial level was 823.2 Joules. Find the height h.

   Show all your work and box the final answer.

6. (10 points) In the year 2020 (which we take to be $t = 0$) there are 5000 wolves in a certain forest.

   In the absence of hunting, the wolf population would increase at the rate of 1% per year. However, hunters are killing wolves at the steady rate of 100 wolves per year.

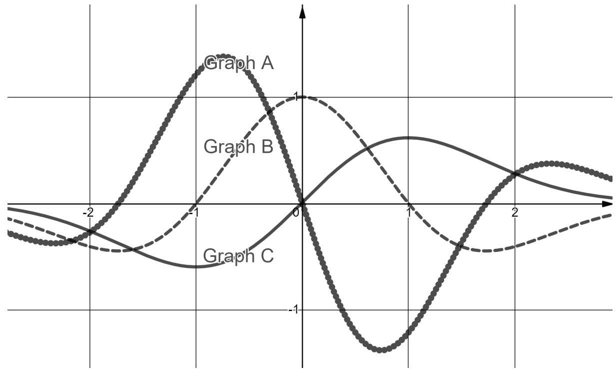
   Let $W(t)$ represent the wolf population in this forest, t years after 2020.

   (a) Write a differential equation that $W(t)$ satisfies.

   (b) Solve this differential equation to find a formula for $W(t)$ in terms of t. Show all steps.

   (c) In what year will the entire wolf population be exterminated from this forest area? Round your answer to the nearest year. Your answer should be some year in this century.

7. (10 points) (a) The following picture shows the graphs of three functions, labeled A, B, and C.



For each of (i)-(iii) below, list all correct answers among the graphs A, B, C, or state "none". No need to justify.

i. Which of A-C could be the graph of an <u>antiderivative</u> of Graph A?

ii. Which of A-C could be the graph of an <u>antiderivative</u> of Graph B?

iii. Which of A-C could be graph of an <u>antiderivative</u> of Graph C?

(b) Which of the following expressions $y = f(x)$ are solutions of the differential equation:

$$\frac{dy}{dx} = e^{x^2} \text{ satisfying } f(1) = 2?$$

Circle the correct ones, and cross out the incorrect ones. For the ones that you crossed out, show which of the conditions it fails to satisfy: $\frac{dy}{dx} = e^{x^2}$, $f(1) = 2$, or both.

(A) $y = \int_1^x e^{t^2}\,dt$

(B) $y = \int_1^x e^{t^2}\,dt + 2$

(C) $y = \int_1^{x^2} e^t\,dt$

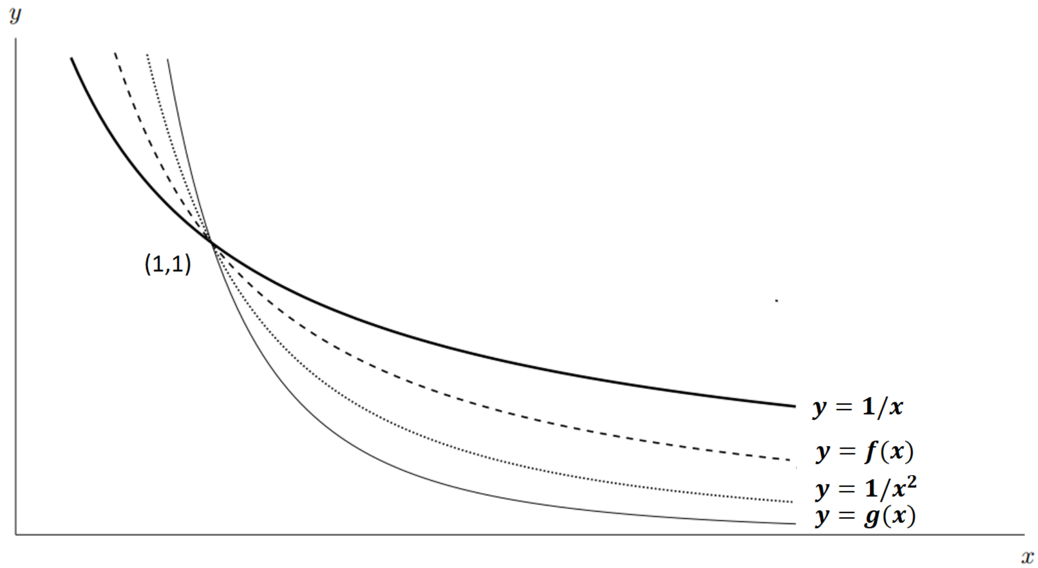
(D) $y = \int_1^x (e^{t^2} + 2)\,dt$

(E) $y = \frac{1}{2x}e^{x^2} - \frac{1}{2}e + 2$

8. (10 points) Evaluate the following integral. Show all steps and use the methods of this class. Credit will depend on clarity and completeness of the solution steps shown.

$$\int \operatorname{arcsec}\left(\sqrt{x^2+4x+5}\right)\,dx$$

9. (10 points) Consider the graphs below, depicting four positive, continuous, and decreasing functions. The functions cross only at at the point $(1,1)$.



Use the graph and the Comparison Test to determine whether the following improper integrals converge or diverge. Circle the appropriate answer. If there is not enough information to determine convergence or divergence, circle "not enough information". **Justify your answers.**

(a) $\displaystyle\int_1^\infty g(x)dx$

CONVERGES DIVERGES NOT ENOUGH INFORMATION

(b) $\displaystyle\int_1^\infty \frac{1}{g(x)}dx$

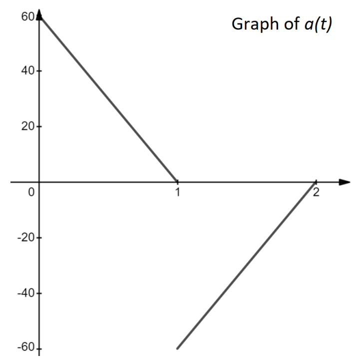
CONVERGES DIVERGES NOT ENOUGH INFORMATION

(c) $\displaystyle\int_1^\infty \sqrt{f(x)}dx$

CONVERGES DIVERGES NOT ENOUGH INFORMATION

10. (10 points) In a 2-second laboratory experiment, a particle moves in a straight line, while its acceleration is manipulated by a force field. The resulting acceleration function (in m/s^2) is a multi-part function, whose expression and graph are shown below.

$$a(t) = \begin{cases} 60(1-t) & 0 \leq t < 1 \\ 60(t-2) & 1 \leq t < 2 \end{cases}$$



Assume that the particle begins at rest at $t = 0$ seconds.

(a) Compute the expression in terms of t for the velocity of the particle from $t = 0$ to $t = 1$ seconds.

(b) Compute the expression in terms of t for the velocity of the particle from $t = 1$ to $t = 2$ seconds.

(c) Compute the average velocity at which the particle was moving from $t = 0$ to $t = 2$ seconds. Show work.