

Your Name

Your Signature

Student ID #

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Quiz Section

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Professor's Name

TA's Name

- Turn off and stow away all cell phones, watches, pagers, music players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- In order to receive credit, you must **show your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas in the table below, without deriving them. **Show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$	2. $\int \frac{1}{x} dx = \ln x $
3. $\int e^x dx = e^x$	4. $\int b^x dx = \frac{b^x}{\ln b}$
5. $\int \sin x dx = -\cos x$	6. $\int \cos x dx = \sin x$
7. $\int \sec^2 x dx = \tan x$	8. $\int \csc^2 x dx = -\cot x$
9. $\int \sec x \tan x dx = \sec x$	10. $\int \csc x \cot x dx = -\csc x$
11. $\int \sec x dx = \ln \sec x + \tan x $	12. $\int \csc x dx = \ln \csc x - \cot x $
13. $\int \tan x dx = \ln \sec x $	14. $\int \cot x dx = \ln \sin x $
17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$	18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

- Place a box around your answer to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- All exam pages are double-sided except for this cover page and the last page. You may use the blank sides for extra room if needed but if you want us to grade these spare pages clearly **indicate in the problem area** that your work is on the back of the cover page or on the blank pages at the end of the exam.
- This exam has 10 problems on 10 pages. When the exam starts, make sure that your exam is complete. Good luck!

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1. (10 points) Evaluate the following integrals. Show your work. Simplify and box your answers.

(a) $\int x^2(1+x)^{2022} dx$

(b) $\int \frac{x}{\sqrt{x^2 - 6x + 10}} dx$

2. (10 points) Evaluate the following integrals. Show your work. Simplify and box your answers.

(a) $\int_0^{\sqrt{\pi}} x^3 e^{x^2} dx$

(b) $\int_{(\pi/6)^{10}}^{(\pi/3)^{10}} \frac{(1 + \tan^2(x^{0.1}))^{1/2}}{x^{0.9}} dx$

3. (10 points) In this question, you do not need to show work or to justify your answers.

Parts (a) and (b) below are not related.

(a) Suppose $f(x)$ is a continuous function defined for all real numbers x , and $A(x) = \int_0^x f(t) dt$.

i. For what values of x is the graph of the curve $y = A(x)$ increasing?

Choose one of the following:

- where $f(x) > 0$
- where $f'(x) > 0$
- where $f''(x) > 0$

ii. For what values of x is the graph of the curve $y = A(x)$ concave up?

Choose one of the following:

- where $f(x) > 0$
- where $f'(x) > 0$
- where $f''(x) > 0$

(b) A particle is moving along a line with velocity $v(t) = t^2 - 8t + 15$.

During which of the following time intervals is the displacement of the particle during that interval equal to the total distance traveled during that time interval?

Choose all that apply.

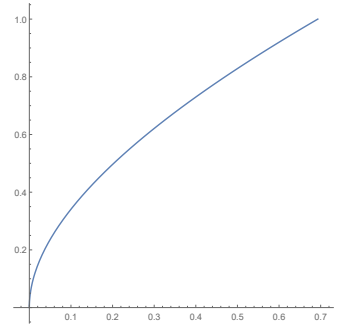
- $1 \leq t \leq 3$
- $2 \leq t \leq 4$
- $4 \leq t \leq 6$
- $5 \leq t \leq 7$

4. (10 points) A region is bounded by the function

$$x = y \ln(y + 1),$$

the x -axis and the line $x = \ln 2$. Note that when $y = 1$, $x = \ln 2$.

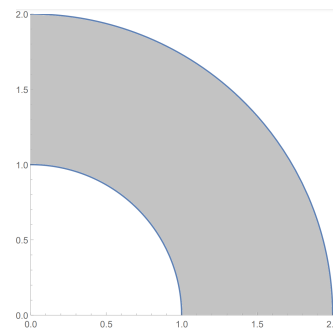
Find the volume of the solid of revolution obtained by rotating the region about the x -axis. Give the answer in exact form or as a decimal number with 5 significant digits.



5. (10 points) Suppose A is the annulus with inner radius 1 and outer radius 2, centered at $(0,0)$. Let B be the part of A in the first quadrant, as shown in the picture.

Find the center of mass of B , assuming constant density. Give your answer in exact form or in the decimal form with at least 5 significant digits.

Hint: It is OK to use symmetry.

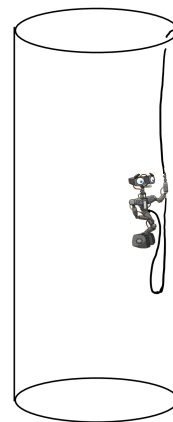


6. (10 points) A rope weighing 0.3 pounds per foot was tied to a robot and it was used to lower the robot into a 30-foot deep well.

The robot will get out of the well by climbing up the rope at a constant speed, with the end of the rope still tied to the robot.

At the beginning of the climb, the robot weighs 20 pounds, including fuel, but it will burn fuel at a constant rate and will lose 3 pounds of fuel during the climb.

Compute the work that the robot will do in climbing up to the top of the well.



7. (10 points) Let R be the region in the first quadrant below the parabola $y = -x^2 + 4x$.

Find the value of $c > 0$ for which the graph of the parabola $y = cx^2$ divides the region R into two subregions of equal area.

Hint: Draw a picture and find the intersection points.

8. (10 points) (a) Set up a definite integral for the arclength of the curve $y = 3x^3$ for $0 \leq x \leq 1$. DO NOT EVALUATE THIS INTEGRAL.

(b) Approximate the integral in part (a) using the Trapezoid Rule with $n = 3$ subintervals. Give your answer in exact form (in terms of square roots, not decimals).

9. (10 points) Find the solution to the differential equation

$$y' = xy(y - 1)$$

that satisfies the initial condition

$$y(0) = -1.$$

Give your solution in explicit form, $y = f(x)$.

10. (10 points) A large vat initially contains 2000 liters of milk with 2% milk fat (by volume). Milk with 4% fat is pumped into the vat at a rate of 20 liters per minute. The milk in the vat is kept thoroughly mixed, and is pumped out of the vat, also at a rate of 20 liters per minute.

(a) What is the percentage of milk fat in the vat after 20 minutes?

(b) How many minutes after the initial time is the percentage of milk fat in the vat equal to 3%?

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