Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and stow away all cell phones, watches, pagers, music players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- In order to receive credit, you must **show your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas in the table below, without deriving them. **Show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$
2. $\int \frac{1}{x}\,dx = \ln|x|$
3. $\int e^x\,dx = e^x$
4. $\int b^x\,dx = \frac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$
8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$
10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln|\sec x + \tan x|$
12. $\int \csc x\,dx = \ln|\csc x - \cot x|$
13. $\int \tan x\,dx = \ln|\sec x|$
14. $\int \cot x\,dx = \ln|\sin x|$
17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

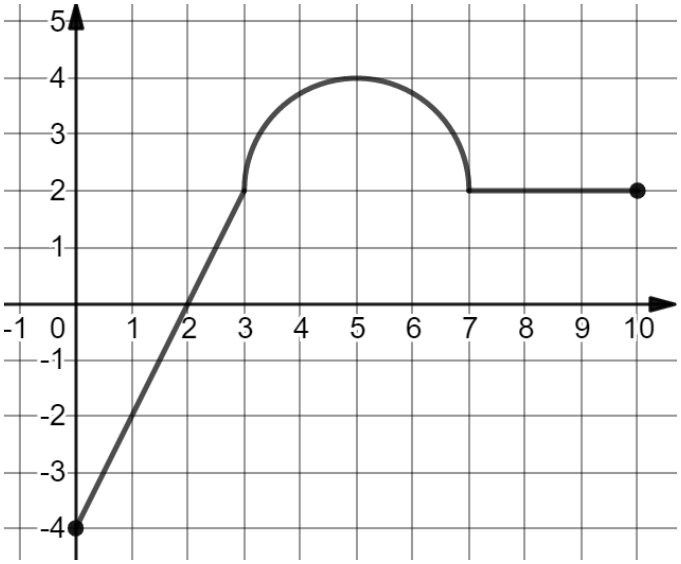
- Place $\boxed{\text{a box around your answer}}$ to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- All exam pages are double-sided except for this cover page and the last page. You may use the blank sides for extra room if needed but if you want us to grade these spare pages clearly **indicate in the problem area** that your work is on the back of the cover page or on the blank pages at the end of the exam.
- This exam has 10 problems on 9 pages. When the exam starts, make sure that your exam is complete. Good luck!

1. (14 points) Evaluate the following integrals. Show your work. Simplify and box your answers.

   (a) (6 points) $\int_0^{2\pi} \sin^2\theta \, d\theta$

   (b) (8 points) $\int \frac{1}{x^3+x^2+x} dx$

2. (10 points) The graph of $y = f(x)$ on $[0,10]$ is shown below. It consists of two line segments and a half circle. Use it to answer the following questions. Show your steps.



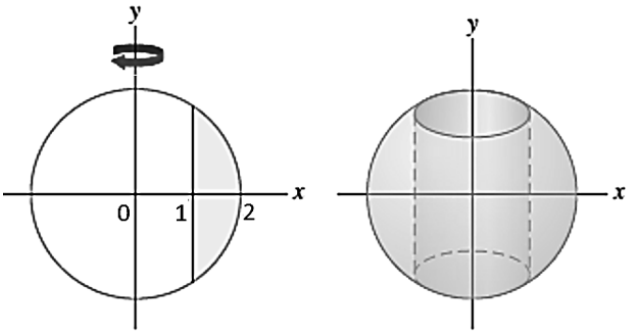
(a) (3 points) Find the average value of f on the interval $[0,3]$.

(b) (2 points) Use the graph and areas to compute $\int_0^{10} |f(x)|\,dx$

(c) (3 points) Define $F(t) = \int_0^t f(x)\,dx$. Evaluate $F'(5)$.

(d) (2 points) Evaluate $\int_3^7 \sqrt{1+[f'(x)]^2}\,dx$. *(Hint: There is a quick way to answer this question!)*

3. (5 points) A "bead" is formed by drilling a hole of radius $r = 1$ cm through the center of a sphere of radius $R = 2$ cm.
   Set up an integral equal to the volume of the resulting bead. **Do not compute the integral.**



4. (5 points) Suppose all we know about some continuous function $g(x)$ is that

$$\frac{1}{x^2} \leq g(x) \text{ for all } x \geq 1.$$

   Circle which of the following statements MUST be true, based on the provided information, and justify the statement(s) that you circled.

   (a) $\int_1^\infty g(x)\,dx$ diverges

   (b) $\int_1^\infty xg(x)\,dx$ diverges

   (c) $\int_1^\infty \frac{g(x)}{x}\,dx$ converges

   (d) $\int_1^\infty g(x)\,dx$ converges

   (e) None of the above

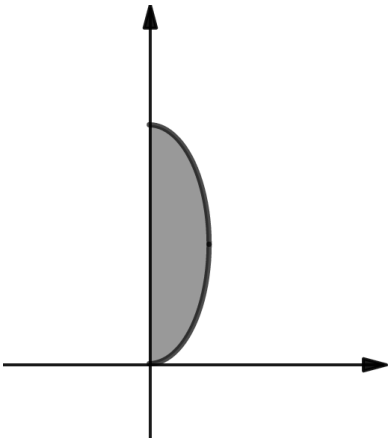
5. (10 points) A rope is 10 m long, has a total mass of 20 kg, and hangs over the edge of a tall building. Recall that $g = 9.8\,m/s^2$.

   (a) (5 points) Compute the work required to pull the entire rope to the top of the building. Include **units**.

   (b) (5 points) Compute the work required to pull just half the rope to the top of the building.

6. Consider the region R in the first quadrant bounded by the y-axis and by the ellipse

$$x^2 + \frac{(y-2)^2}{4} = 1$$



(a) (8 points) Compute the area of this region.

(b) (8 points) Find the coordinates $(\bar{x}, \bar{y})$ of the centroid of this same region.

7. (8 points) Compute the arc length of the curve

$$y = \frac{e^x + e^{-x}}{2}$$

over the interval $0 \leq x \leq 1$.

8. (10 points) The following two differential equations may appear similar but have very different solutions. Solve both, subject to the same initial condition, as indicated. Show your steps and give your final answers in explicit form, $y = y(x)$.

   (a) (5 points) $\dfrac{dy}{dx} = x$, with $y(1) = 2$.

   (b) (5 points) $\dfrac{dy}{dx} = y$, with $y(1) = 2$.

9. (10 points) A container has 75 gallons of liquid in it. At noon, liquid starts being poured into the container at a constant rate of 2 gallons/min. At the same time, liquid starts leaking out of the container through a hole on the bottom, at a rate out that is proportional to the current volume of liquid in the container.

   (a) (3 points) Let $V(t)$ denote the volume of liquid in the container, in gallons, at t minutes past noon. Write down the differential equation and initial value satisfied by $V(t)$. Use k for the constant of proportionality.

   (b) (7 points) Assume the constant of proportionality is $k = 0.2$. Solve the differential equation from part (a) to obtain an explicit expression for $V = V(t)$, and then determine what happens to the volume of the liquid in the container as time goes on (i.e. as $t \to \infty$).

10. Recall that $\int \ln(x)dx = x\ln(x) - x + C$. You may use this without further justification.

(a) (8 points) Compute $\int (\ln x)^2 dx$ and $\int (\ln x)^3 dx$.

(b) (4 points) Let k be any positive integer. Show that:

$$\int (\ln x)^k dx = x(\ln x)^k - k\int (\ln x)^{k-1} dx$$