Math 125 **Final Exam** **Autumn 2019**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and stow away all cell phones, watches, pagers, music players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas # 1-18 in the table from section 7.5 of your textbook (posted on the departmental math 125 website), without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place a box around your answer to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- If you need more room, use the backs of the pages and indicate that you have done so.
- This exam has 9 pages, in addition to this cover sheet. Make sure you have a complete exam.

Question	Points	Score
1	12	
2	14	
3	8	
4	7	
5	21	

Question	Points	Score
6	8	
7	8	
8	10	
9	12	
Total	100	

1. (12 points) Evaluate the following integrals. Show your work. Simplify and box your answers.

   (a) $\int_0^{\pi/4} y\sin(y)\,dy$

   (b) $\int_0^{\pi/6} 16\cos^2(2x)\sin^2(2x)\,dx$

2. (14 points) Answer the following two unrelated questions. Show your work and box your answer.

   (a) Evaluate the integral: $\int \ln(x^2+1)\,dx$

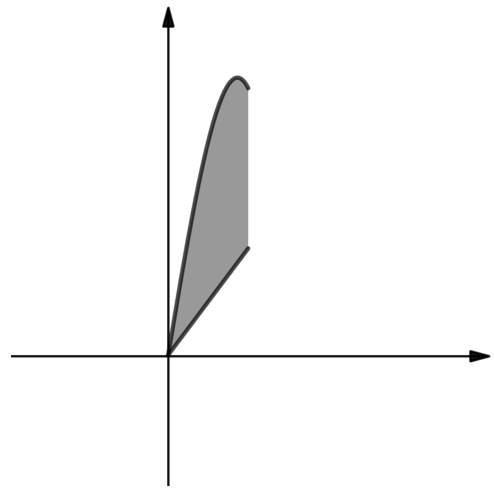
   (b) The acceleration and the initial velocity of a object moving on a straight line are given by:

   $$a(t) = 2t + 6 \;\; m/s^2 \quad \text{and} \quad v(0) = -7 \;\; m/s$$

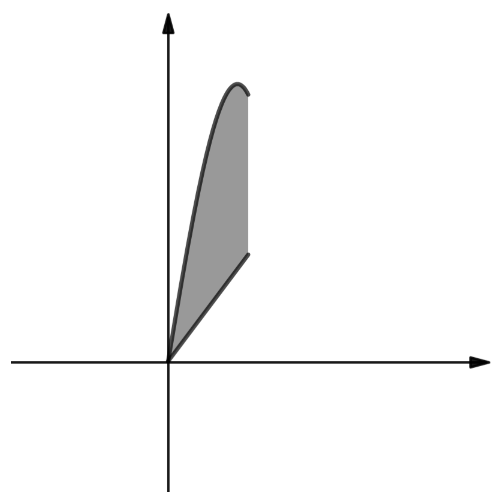
   Find the **<u>total</u> distance** traveled by the particle from $t = 0$ to $t = 2$ seconds.

3. Consider the region enclosed by the graphs $y = 9x - x^3$, $y = 2x$, $x = 0$, and $x = 2$ pictured below.

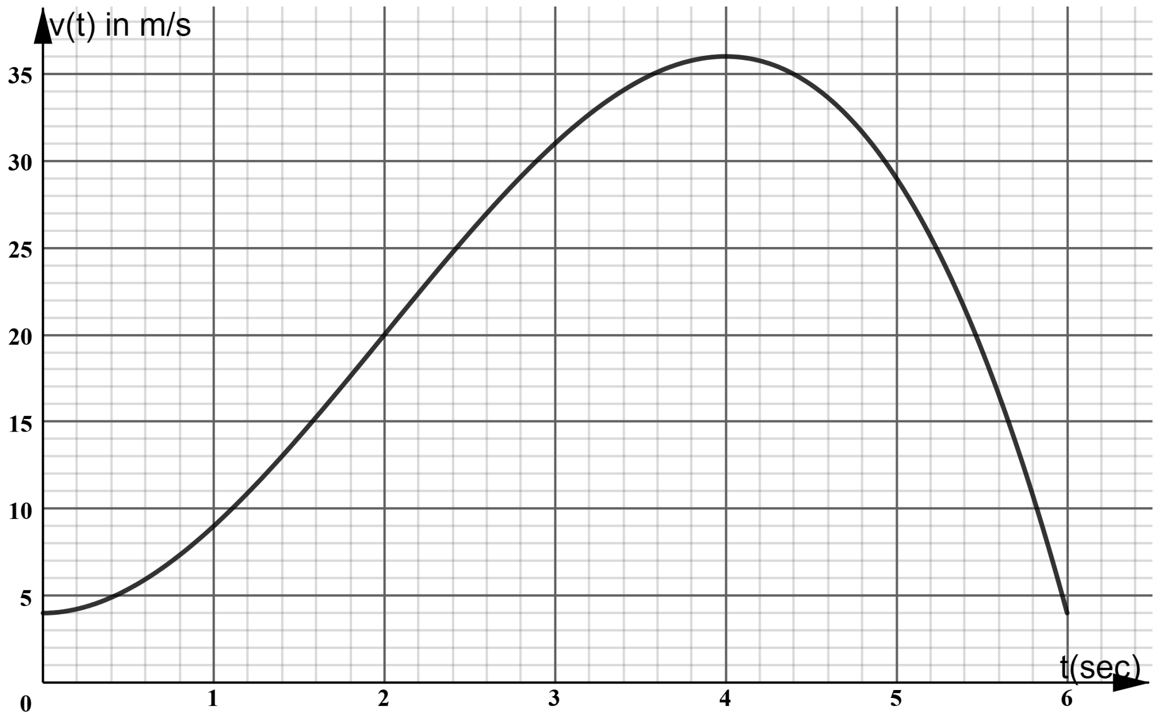
(a) (4 points) **Set up** an integral that represents the volume of the solid formed by rotating this region about the y-axis. (Do not compute the volume).



(b) (4 points) **Set up** an integral that represents the volume of the solid formed by rotating this region about the line $y = -5$. (Do not compute the volume).



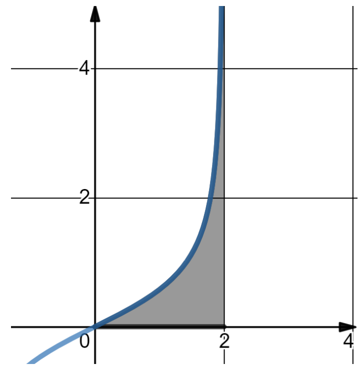
4. (7 points) The graph below shows the instantaneous velocity $v(t)$ (in meters per second) of an object moving along a straight line, as a function of time t (in seconds).



Use Simpsons Rule with $n = 6$ subintervals to approximate the **average velocity** v_{ave} of the object from $t = 0$ to $t = 6$ seconds.

5. Let $\mathcal{R}$ be the region in the first quadrant which is shown below, and it is described by:

$$0 \leq y \leq \frac{x}{\sqrt{4-x^2}}, \qquad 0 \leq x < 2$$

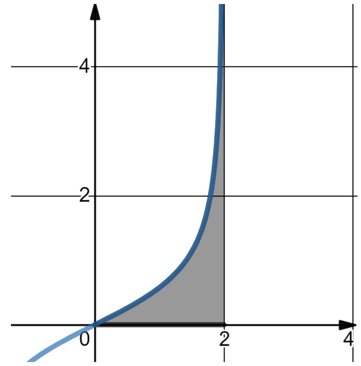


Note that $f(x) = \frac{x}{\sqrt{4-x^2}}$ *has a vertical asymptote; use limits for improper integrals as needed, and determine if they converge or diverge.*

(a) (6 points) Compute the **area** of this region $\mathcal{R}$.

(b) (7 points) Compute the x-coordinate, $\bar{x}$, of its centroid (center of mass).

(*problem continues on the next page*)



(c) (8 points) *Recall the region $\mathcal{R}$ from the previous page, bounded above by* $y = \dfrac{x}{\sqrt{4-x^2}}$, for $0 \leq x < 2$.

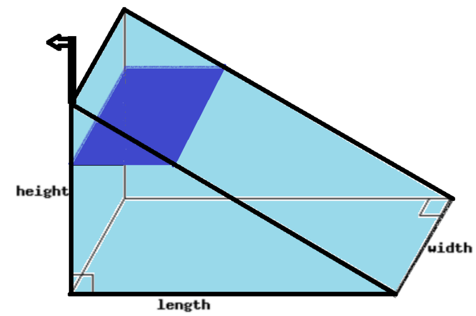
Use limits for improper integrals as needed, and determine if they converge or diverge.

Compute the y-coordinate, $\bar{y}$, of the centroid of $\mathcal{R}$.

6. (8 points) A tank of the shape shown in the picture, with height=7m, length=10m, and width=5m, is full of water. Water weighs 1000 kg/m^3, and the gravitational acceleration is $g = 9.8$ m/s^2.

   Set up (do not evaluate) an integral equal to the work required to pump all the water out of the tank through a spout that is 1 m above the top of the tank.

   Specify the meaning of your variable of integration, either in words or on the picture.



7. (a) (4 points) Write down an integral equal to the **arclength** $L(t)$ of the portion of the curve:

$$y = e^{x^2}, \text{ from } x = 0 \text{ to } x = t.$$

   (b) (4 points) At what rate is $L(t)$ increasing when $t = 1$?

8. (10 points) Find the solution to the differential equation

$$\frac{dy}{dx} = \frac{xy+y}{2\ln(y)}$$

that satisfies the initial condition $y(1) = e^2$. Give your solution in explicit form, $y = f(x)$.

9. A 2000 L tank is full of a mixture of water and salt, with 500 grams of salt initial dissolved in the tank. Fresh water (with NO salt) is pumped into the tank at a rate of 20 L/s. The mixture is kept stirred and is pumped out at a rate of 40 L/s. (This means the tank is losing volume at a rate of 20 - 40 = -20 L/s).

   (a) (1 point) Give the linear function $V(t) = at + b$ for the volume in liters after t seconds.

   (b) (4 points) Let $y(t)$ be the amount of salt in grams in the tank after t seconds. Write down the differential equation AND initial condition satisfied by $y(t)$. Do not solve anything yet.

   (c) (6 points) Solve the differential equation to find $y(t)$. Show work. Simplify and box your answer.

   (d) (1 point) How many grams of salt are left in the tank after 60 seconds? Simplify your answer.