Math 125 **Final Examination** **Autumn 2016**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- You can use only a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place a box around your answer to each question.
  Unless otherwise instructed, leave your answers in exact form.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (10 total points) Evaluate the following definite integrals. Simplify and box your answers.

   (a) (5 points) $\displaystyle\int_0^{\pi/4} \tan^2\theta \sec^4\theta \, d\theta$

   (b) (5 points) $\displaystyle\int_0^1 x^2 \arcsin(x) \, dx$

2. (10 points) Evaluate the following indefinite integrals.

   (a) (5 points) $\int \sqrt{10x - x^2}\, dx$

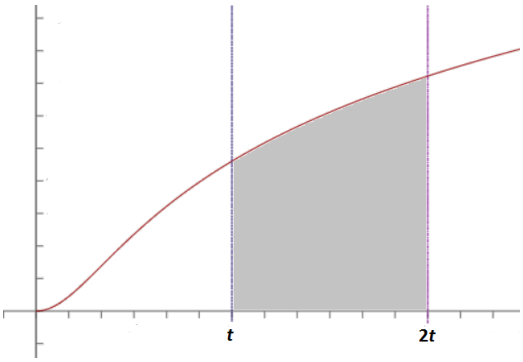
   (b) (5 points) $\int \frac{x-1}{x^3+x}\, dx$

3. (10 points) Consider the infinite region in the first quadrant of the xy-plane, above the line $y = 1$, and to the left of the curve

$$y = \frac{1}{\sqrt{x}}$$

Rotate this region about the y-axis, and determine whether the volume of the resulting solid is finite or infinite. If it is finite, find the volume. If it is infinite, explain why.

4. (10 total points) The figure on the right shows a region bounded to the left by the line $x = t$, to the right by $x = 2t$, on the top by the curve $y = \ln(x^2 + 1)$, and on the bottom by the x-axis.



(a) (2 points) Set up an integral for the area $A(t)$ of this region. DO NOT EVALUATE the integral.

(b) (4 points) Compute $A'(1)$

(c) (4 points) Set up an integral for the arc length $L(t)$ of the top boundary of this region (that is, the arc length of the curve $y = \ln(x^2 + 1)$, $t \leq x \leq 2t$). DO NOT EVALUATE the integral.

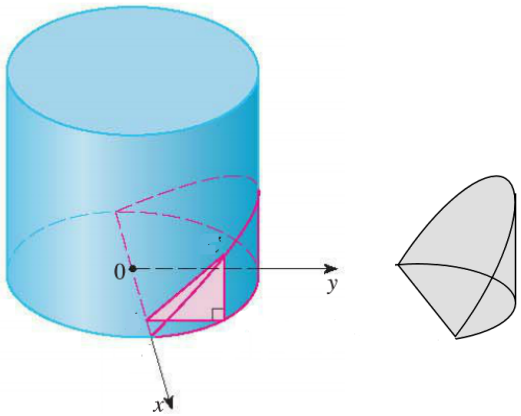
5. (10 points) The curves:

$$y = x^3 \qquad \text{and} \qquad y = 4x$$

enclose two regions in the plane. Find the total area of these regions.

6. (10 points) A wedge is cut out of a right cylinder of radius 2 by two planes. One plane is horizontal, perpendicular to the axis of the cylinder. The other plane intersects the first at an angle of 30° along a diameter of the cylinder.

   Compute the volume of the wedge.



7. (10 total points) A calculus student lifts a bag of sand, beginning at time $t = 0$, at a constant rate of 0.20 meters per second, from the ground to a height of 2 meters above ground. The initial mass of the sand in the bag is 10 kilograms (the mass of the bag is negligible) but sand drains from a hole in the bottom of the bag at the variable rate of $\frac{1}{1+t}$ kilograms per second. Recall that the gravitational acceleration is $g = 9.8\ m/s^2$.

   (a) (4 points) Find the mass $m(t)$ of sand in the bag at time t seconds.

   (b) (6 points) Set up an integral for the work done in lifting the sand. DO NOT EVALUATE the integral.

8. (10 points) Find the solution of the differential equation below that satisfies the given initial condition. Write your solution y as an explicit function of x, that is, your answer should be expressed in the form $y = f(x)$.

$$y' = xe^x + xy^2e^x \qquad\qquad y(0) = 0$$

9. (10 points) A tank contains 100 liters of fresh water. Water containing s grams of salt per liter enters the tank at the rate of 10 liters/minute, and the well-mixed solution leaves at the same rate.

   (a) (3 points) Let $y(t)$ denote the amount of salt in the tank at time t minutes. Write down a differential equation for $y(t)$. (This equation will contain the constant s.)

   (b) (7 points) Suppose that after 20 minutes the concentration of salt in the tank is 3 grams/liter. Compute s. Leave your answer in exact form.

10. (10 total points) Consider the region bounded by $x = 1$, $x = 10$, $y = \dfrac{1}{x}$, and $y = \dfrac{1}{2x}$.

(a) (8 points) Find the centroid (center of mass) of this region.

(b) (2 points) Does the centroid lie inside the region? Justify your answer.