

Your Name

Your Signature

Student ID #

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Quiz Section

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Professor's Name

TA's Name

- This exam is closed book. You may use one  $8.5'' \times 11''$  sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Give your answers in exact form (for example  $\frac{\pi}{3}$  or  $5\sqrt{3}$ ).
- **No calculators** of any kind are allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet.**
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (10 total points) Evaluate the following definite integrals.

(a) (5 points)  $\int_0^{\pi/10} \sin^2(5x) \cos^3(5x) dx$

(b) (5 points)  $\int_1^3 \frac{x^2}{x^2 - 2x + 5} dx$

2. (10 total points) Evaluate the following indefinite integrals.

(a) (5 points)  $\int \frac{dx}{x^2\sqrt{4+x^2}}$

(b) (5 points)  $\int \frac{x^{1/4} + 1}{x(x^{1/4} - 2)^2} dx$

3. (10 points) Determine whether the following improper integral converges and, if so, evaluate it. Justify your answer.

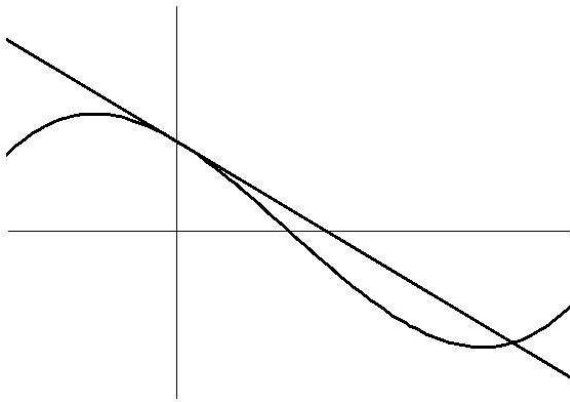
$$\int_1^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$$

4. (10 points) The position  $s(t)$  of an object moving along a straight line is given by the formula

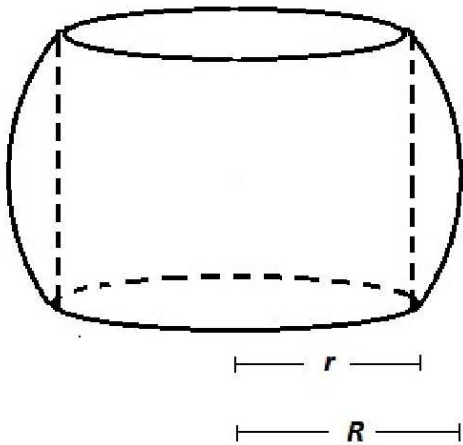
$$s(t) = \int_{t^2}^t e^{\sin x} dx.$$

Find a formula for the acceleration  $a(t)$  of the object as a function of time  $t$ .

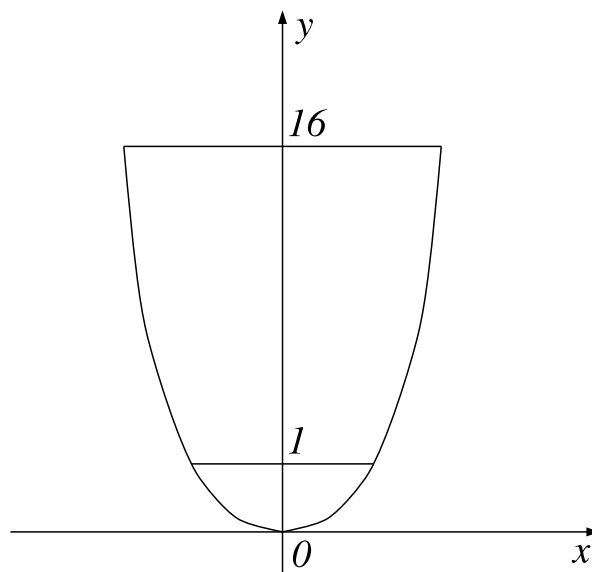
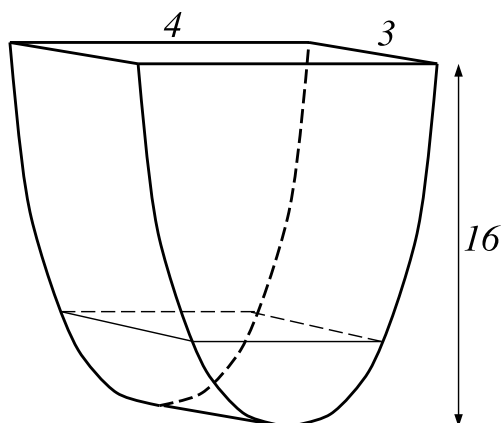
5. (10 points) In the graph below, the line is tangent to the curve  $y = x^3 - 3x^2 - 6x + 8$  at the point on the y-axis. Find the area enclosed between the curve and the line as shown in the figure.



6. (10 points) A manufacturer drills a round hole of radius  $r$  through the center of a metal sphere of radius  $R$ . Find the volume of the remaining metal “bead.”



7. (10 points)



A tank (shown in the figure on the left) is 16 feet high, with an open rectangular top of width 3 feet and length 4 feet. Each horizontal cross-section of the tank is a rectangle of fixed width 3 feet and length that changes with height. The figure on the right shows the front face of the tank, which has the shape of the function  $y = x^4$  for  $-2 \leq x \leq 2$ .

Initially, there is fluid in the tank up to a height of 1 foot. The fluid weighs  $15 \text{ lb/ft}^3$ . How much work does it take to empty the tank by pumping all of the fluid to the top of the tank?



8. (10 total points)

(a) (4 points) Write a definite integral for the arclength  $L$  of the graph of  $y = x^2$  from  $x = 1$  to  $x = 3$ .  
DO NOT EVALUATE THE INTEGRAL.

(b) (6 points) Use Simpson's Rule with  $n = 4$  subintervals to approximate the definite integral in part (a). Give your answer in exact form.

9. (10 points) Find the solution of the initial value problem

$$\frac{dy}{dt} = t \sin(t) \cos^2(y), \quad y(0) = \frac{\pi}{4}.$$

Give your answer in the form  $y = f(t)$ .

10. (10 total points) An advertising company introduces a new product to Seattle. Let  $P = P(t)$  be the number of people *in thousands* who are aware of this new product at time  $t$  in days. Seattle has a total population of 700 thousand. Initially, no one has heard of the product.
- (a) (3 points) Set up a differential equation for  $P$  if it increases at a rate proportional to the number of people in Seattle still unaware of the product. Also state the initial condition for  $P$ .
- (b) (5 points) After 20 days, 140 thousand people have heard of the product. Solve the differential equation for  $P(t)$ , and also determine the exact value of any constants in your solution.
- (c) (2 points) How long does it take before half the population has heard of the product? Give your answer in exact form.